2 Solve by Assuming

Problems introduced in this chapter are most commonly solved using tables, which can be tedious at times. Instead, we attempt to solve problems of this nature by asking, "If all were ..."

EXAMPLES

1 A farmer has 36 chickens and rabbits. There are 96 legs altogether. How many chickens does the farmer have?
How many rabbits does the farmer have?

Solution:

Method 1: Solve by Assuming

If all are chickens, there will be $36 \times 2 = 72$ legs.

$$96 - 72 = 24$$

All the rabbits have been counted as if they are chickens.

$$4 - 2 = 2$$

The difference in the number of legs between a chicken and a rabbit is 2.

$$24 \div 2 = 12 \text{ rabbits}$$
$$36 - 12 = 24 \text{ chickens}$$

Method 2: Solve by Assuming

If all are rabbits, there will be $36 \times 4 = 144$ legs.

$$144 - 96 = 48$$

There will be an extra 48 legs.

$$4 - 2 = 2$$

The difference in the number of legs between a rabbit and a chicken is 2.

$$48 \div 2 = 24 \text{ chickens}$$
$$36 - 24 = 12 \text{ rabbits}$$

The farmer has **24** chickens and **12** rabbits.

2 A cashier collected \$312 from the sale of 50 tickets. An adult ticket cost \$8. A child ticket cost \$4. How many tickets of each type did the cashier sell?

Solution:

Method 1: Solve by Assuming

If all were adult tickets, $50 \times \$8 = \$400$ would be collected.

$$\$400 - \$312 = \$88$$

There would be an extra \$88.

$$\$8 - \$4 = \$4$$

The price difference between an adult ticket and a child ticket was \$4.

$$\$88 \div \$4 = 22 \text{ child tickets}$$
$$50 - 22 = 28 \text{ adult tickets}$$

Method 2: Solve by Assuming

If all were child tickets, $50 \times \$4 = \$200$ would be collected.

$$\$312 - \$200 = \$112$$

There would be a shortfall of \$112.

$$\$8 - \$4 = \$4$$

The price difference between an adult ticket and a child ticket was \$4.

$$\$112 \div \$4 = 28 \text{ adult tickets}$$
$$50 - 28 = 22 \text{ child tickets}$$

The cashier sold **28** adult tickets and **22** child tickets.

3 There are 30 questions in a mathematics competition. 5 marks are awarded for each correct answer and 2 marks are deducted for each incorrect answer. If Colin scores 122 marks for the mathematics competition, how many incorrect answers does he give?

Solution:

If Colin gets all the answers correct, he will get $30 \times 5 = 150$ marks.

$$150 - 122 = 28$$

There is an extra 28 marks.

$$5 + 2 = 7$$

The difference in marks between a correct answer and an incorrect answer is 7 marks.

$$28 \div 7 = 4 \text{ incorrect answers}$$

He gives **4** incorrect answers.

4 5 similar basketballs and 6 similar volleyballs cost \$325. A basketball costs \$10 more than a volleyball. What is the price of a basketball?

Solution:

Method 1: Solve by Assuming

$$\$325 + 6 \times \$10 = \$385$$

If all are basketballs, the total price will be \$385.

$$5 + 6 = 11$$

The total number of basketballs will then be 11.

$$385 \div 11 = \$35$$

Method 2: Solve by Assuming

$$\$325 - 5 \times \$10 = \$275$$

If all are volleyballs, the total price will be \$275.

$$5 + 6 = 11$$

The total number of volleyballs will then be 11.

$$\$275 \div 11 = \$25$$
$$\$25 + \$10 = \$35$$

The price of a basketball is **\$35**.

5 Mr Brannon has 58 pieces of two-dollar, five-dollar and ten-dollar notes altogether in his wallet. The total value of these notes is \$322. The number of two-dollar and ten-dollar notes is the same. How many notes of different denominations does he have?

Solution:

We 'create' a new denomination, which is a $(\$10 + \$2) \div 2 = \$6$ note.

Method 1: Solve by Assuming

If all are six-dollar notes, the total value will be $58 \times \$6 = \$348$.

$$\$348 - \$322 = \$26$$

The difference in the total value of all the notes is \$26.

$$\$6 - \$5 = \$1$$

The difference in value between a six-dollar note and a five-dollar note is \$1.

\$26 ÷ \$1 = 26 pieces of five-dollar notes
58 – 26 = 32 pieces of two-dollar and ten-dollar notes
32 ÷ 2 = 16 pieces for each two-dollar and ten-dollar notes

Method 2: Solve by Assuming

If all are five-dollar notes, the total value will be $58 \times \$5 = \$290$.

$\$322 - \$290 = \$32$

The difference in the total value of all the notes is \$32.

$\$6 - \$5 = \$1$

$\$32 \div \$1 = 32$ pieces of two-dollar and ten-dollar notes

$32 \div 2 = 16$ pieces for each two-dollar and ten-dollar notes

$58 - 32 = 26$ pieces of five-dollar notes

He has **16** two-dollar notes, **26** five-dollar notes and **16** ten-dollar notes.

PRACTICE

1 A farmer has 45 chickens and rabbits. There are 140 legs altogether. How many chickens does the farmer have? How many rabbits does the farmer have?

2 A spider has 8 legs. A dragonfly has 6 legs. 28 spiders and dragonflies have 200 legs altogether. How many spiders are there? How many dragonflies are there?

3 Andy spent $55 in all to purchase 20 pieces of two-dollar and five-dollar stamps. How many two-dollar stamps did he buy? How many five-dollar stamps did he buy?

4 There are 40 cars and motorcycles in a park. There are 116 wheels altogether. How many cars are there? How many motorcycles are there?

5 An adult ticket to a concert cost $35 and a concession ticket cost $18. Mr Walter paid $598 in all for 20 such tickets. How many adult tickets did he buy? How many concession tickets did he buy?

6 Jeff has 20 pieces of five-dollar and ten-dollar notes in his wallet. The total value of these notes is $125. How many notes of each denomination does he have?

7 A science quiz consists of 15 questions. 2 marks are awarded for every correct answer and 1 mark is deducted for every wrong answer. Kelly scores 21 marks for the science quiz. How many questions does she answer correctly?

8 There are 30 questions in a mathematics competition. 5 marks are awarded for each question answered correctly and 3 marks are deducted for each wrong answer. Rena scores 126 marks for the mathematics competition. How many questions does she get wrong?

9 A mathematics quiz consists of 30 questions. The first 20 questions are worth 4 marks each. The last 10 questions are worth 7 marks each. No marks will be deducted for each wrong answer. Justin scores 124 marks for the mathematics quiz. How many of the first 20 questions and how many of the last 10 questions does he answer wrongly?

10 Mr George spent $375 in all for 5 similar tables and 6 similar chairs. Each table cost $20 more than each chair. What was the price of a table? What was the price of a chair?

11 Mr Gretzky spent $390 on 5 similar basketballs and 4 similar volleyballs. Each basketball cost $15 more than each volleyball.
What was the price of a basketball and the price of a volleyball?

12 Wayne has 20 pieces of notes. The types of denominations are $2, $5 and $10. The number of $2 notes and $10 notes is the same. The total value of these notes is $110. How many pieces of each denomination does he have?