10 Solve Using Table or Drawing

Deployment of different strategies for different mathematical problems helps to alleviate the laborious process of working through them. Solving a mathematical problem using a table or drawing is another useful resource to be tapped into.

Tables and drawings are simple yet powerful tools for solving mathematical problems such as logic, working backwards, speed, etc.

The beauty of solving mathematical problems using table or drawing is manifold.

A well-sketched drawing sometimes will present the solution by itself almost instantly. A table, on the other hand, makes problem-solving systematic and allows us to do checking.

Furthermore, the tools of drawing or table help us translate a question into visual diagrams or images. This is especially true when we work on the speed problems.

Drawing is also a useful tool when we work on mathematical problems related to working backwards.

EXAMPLES

1 $\frac{5}{7}$ of number A is the same as $\frac{1}{4}$ of number B.

What is the minimum sum of numbers A and B?

***Solution*:**

Method 1: Solve Using Drawing

A

B

1 part of B → 5 parts of A

$4 \times 5 + 7 = 27$

Method 2: Solve by Algebra

$$\frac{5}{7}A = \frac{1}{4}B$$

$$A = \frac{1}{4}B \times \frac{7}{5}$$

$$= \frac{7}{20}B$$

$$A + B = \frac{7}{20}B + B$$

$$= \frac{27}{20}B$$

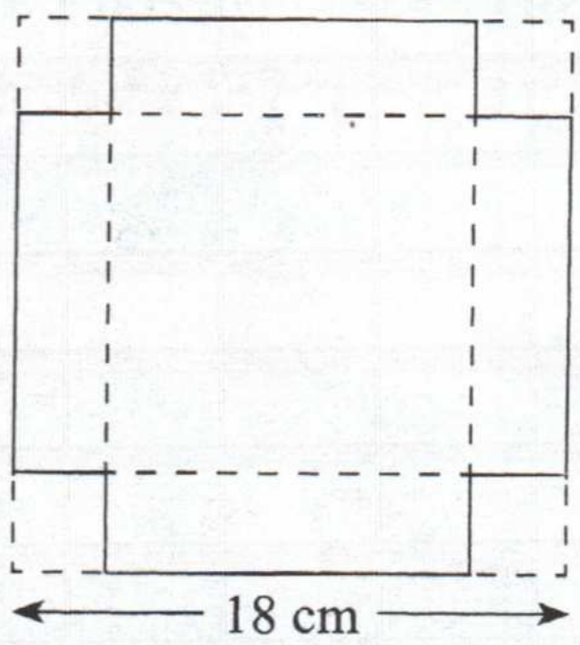
When $B = 20$,

$$\frac{27}{20} \times 20 = 27$$

$$A + B = 27$$

The minimum sum of numbers A and B is **27**.

2 The four corners of a square piece of paper of side 18 cm are cut off as shown below. The piece of paper is then folded along the dotted lines into a tray. What must be the length of each of the four small squares so that the volume of the tray is the maximum? (Consider only whole numbers.)



***Analysis*:** Make a table that will allow us to check systematically.

***Solution*:**

side	1 cm	2 cm	3 cm	4 cm
base	16 cm	14 cm	12 cm	10 cm
volume	$16 \times 16 \times 1$ $= 256\ cm^2$	$14 \times 14 \times 2$ $= 392\ cm^3$	$12 \times 12 \times 3$ $= 432\ cm^3$	$10 \times 10 \times 4$ $= 400\ cm^3$

The length of each of the four small squares must be **3 cm** so that the volume of the tray is the maximum.

3 Three friends happened to wash their cars on 24th of June, a Sunday.

Alan: I wash my car every other day.
Brian: I wash my car every three days.
Charles: I wash my car every four days.
All of them will not wash their cars on Fridays.
They will do it on Saturdays instead.
Which day of the week will it be the next time they wash their cars on the same day?

***Analysis*:** Making a table will allow us to check systematically.

***Solution*:**

	Sun	M	T	W	Th	F	S	Sun	M	T	W	Th	F	S	Sun	M
Alan	✓		✓		✓	×	✓		✓		✓		×	✓		
Brian	✓			✓		×	✓			✓			×	✓		
Charles	✓				✓	×			✓				×	✓		

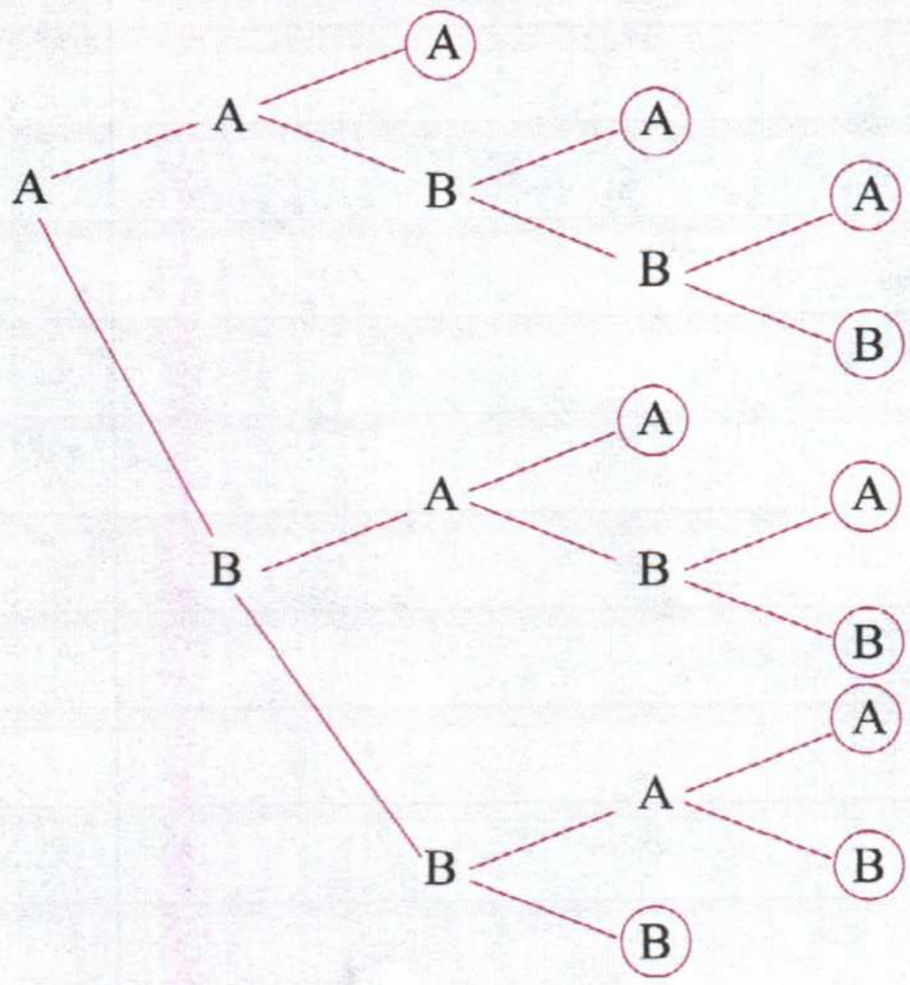
It will be a **Saturday** the next time they wash their cars on the same day.

4 The two finalists in a Snooker League championship will continue to play until one player wins a total of three games. How many ways are there for any finalist to win the championship?

***Analysis*:** There are two scenarios: Finalist A takes the first game, or Finalist B takes the first game.

***Solution*:**

If Finalist A takes the first game,



There are 10 possible results if A takes the 1st game.

$10 \times 2 = 20$ ways

There are **20** ways for any finalist to win the championship.

PRACTICE

1 There are three groups of red and blue marbles.
Each group has the same number of marbles.
The number of red marbles in the first group is the same as the number of blue marbles in the second group.
The number of red marbles in the third group is $\frac{2}{5}$ of the total number of red marbles in all the three groups.
What is the fraction of blue marbles in all the three groups of marbles?

2 156 marbles were to be given to A, B and C.
At first, A got the most number of marbles and C got the least number of marbles.
A then gave some marbles to B and C that doubled the number of marbles they had plus another 4 marbles each.
B also gave some marbles to A and C that doubled the number of marbles they had plus another 4 marbles each.
Finally, C gave A and B some marbles that were 4 more than doubled the number of marbles they had. Now A, B and C had the same number of marbles.
How many marbles did each of them get at first?

3 Jonathan and Samuel saved \$712 altogether.

$\frac{1}{4}$ of Jonathan's savings was \$48 more than $\frac{1}{6}$ of Samuel's savings.

How much did each boy save?

4 Whole numbers 1 to 10 are randomly arranged in the following manner:

10 3 7 4 6 5 8 9 1 2

How many times will you need to adjust the numbers so that it will become

1 2 3 4 5 6 7 8 9 10 ?

(Assume each number can only cross one number from its right at one time.)

5 Suppose a coach left Calgary for Toronto every 4 hours. Another coach left Toronto for Calgary every 4 hours at the same time. It took the coaches 48 hours to arrive at their destinations. How many coaches would an eastbound coach come across before its arrival at the destination?

6 Machines A and B were supposed to produce 560 spare parts altogether. In the end, the two machines jointly produced 656 spare parts. If Machine A produced $\frac{1}{5}$ more spare parts than scheduled and Machine B produced $\frac{3}{20}$ more spare parts than scheduled, how many spare parts was each machine supposed to produce at first?

7 Three football teams played exactly one match against one another in the North Zone Football League. The table below shows the results of each team after the matches.

Team	Win	Loss	Draw	Goals Scored	Goals Conceded
A	2	0	0	6	1
B	0	1	1	2	4
C	0	1	1	3	6

What was the result of each match?

8 A cow produces a calf at the beginning of every year. The young cow will become productive and conceive in the fourth year and give birth to another calf the beginning of every year, starting from the following year. How many cows will there be in the eighth year?

9 When Jeremy divided his marbles into 3 groups of the same number, the remainder was 2. When he again divided his marbles from 2 of the groups into 3 groups of the same number, the remainder was still 2. Lastly, he arranged 2 groups of marbles from the previous division into 3 equal groups of the same number, and the remainder was still 2. How many marbles did Jeremy have at first?